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Comments on the Scalar-Tensor Theory†

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Abstract

Scalar-tensor theories are discussed as encompassing three classical long-range fields, including the electromagnetic field. In order to shed additional light on the restrictive assumptions made by Dicke concerning the coupling of the scalar field with matter, the ponderomotive laws of a scalar-tensor theory are constructed free of approximations in the form of integral laws. The integrals are extended over two- and three-dimensional domains that lie entirely in empty space but surround the regions containing matter; as for the latter, the vacuum field equations are not required to hold, but no further assumptions are made. It turns out that the gradient of the incident scalar field will contribute to the rate of change of the mass and linear momentum of a 'particle' an amount proportional to that particle's scalar-field source strength, which in turn is an arbitrary function of time, unless Dicke's special restriction is imposed. To this extent the motion of a test particle is indeterminate, contrary to experience.

1. *Survey of Scalar-Tensor Theories*

Recent experimental work by Dicke and members of his group (Dicke & Goldenberg, 1967) has rekindled interest in the type of theories that he calls scalar-tensor theories and which historically are based on the five-dimensional unitary field theories by Kaluza (1921) and their modifications by Einstein and Bergmann, Jordan, Thiry, and Dicke's group.‡ In the original paper by Kaluza the unification of gravitation and electrodynamics was achieved by the postulation of a five-dimensional pseudo-Riemannian manifold, restricted by the requirement that there exist a congruence of isometric geodesics. Kaluza's theory leads to a geometry with precisely the same degrees

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‡ References to the literature will be found in Dicke and Goldenberg (1967) and also in Tonnelat, M. A. (1965), *Les Théories Unitaires de l'Électromagnétisme et de la Gravitation*, pp. 180 and 219 f. Gauthier-Villars, Paris.

of freedom characterizing the conventional Einstein-Maxwell theory. If Kaluza's condition is relaxed, so that there exists a congruence of isometric curves (which need not be geodesics), then there emerges a fifteenth field variable, which in four-dimensional terminology is a scalar (Bergmann, 1948).

The task of constructing dynamical laws involving conventional gravitational and electromagnetic potentials, and a scalar as well, appears at first sight to present unlimited possibilities. Their number is considerably reduced if one agrees (a) that the field equations are to be derivable from a least-action principle, and (b) that they be of no higher than the second differential order. The search is then reduced to the consideration of appropriate Lagrangians.

If it were not for the scalar, there would exist only three scalar densities of the requisite differential order, $|g|^{1/2}R$, $|g|^{1/2}$, and $|g|^{1/2}M$. The assumed existence of an additional potential, a scalar, permits the multiplication of the three scalar densities above by arbitrary functions of that scalar (which will be designated by the symbol φ from now on), and the construction of one additional term in the Lagrangian, the square of the gradient of the scalar. The most general Lagrangian compatible with the requirements listed above is of the form

$$L = |g|^{1/2}[f_1(\varphi)\,R + f_2(\varphi)\,M + f_3(\varphi)\,g^{\mu\nu}\,\varphi_{,\mu}\,\varphi_{,\nu} + f_4(\varphi)] \tag{1.1}$$

R is the (four-dimensional) curvature scalar, and M the Maxwell scalar. This Lagrangian is a scalar density with respect to four-dimensional curvilinear coordinate transformations, and invariant with respect to gauge transformations.

The number of arbitrary functions in the expression (1.1) may be reduced further. Redefinition of the metric tensor field of the kind

$$g_{\mu\nu} = a(\varphi)\,g_{\mu\nu}{}^* \tag{1.2}$$

depending on the choice of a, makes it possible to replace f_1 by a constant (or, if preferred, by φ). One might think of doing the same with the electromagnetic potentials, but the gauge group leads to a unique definition of the electric field as that curl that is gauge-invariant. One may, however, replace the scalar potential φ by some function of φ, $b(\varphi)$, so as to make f_3 constant. In so doing one has exhausted all the recalibrations of field variables available.

The last term in the expression (1.1) leads to cosmological terms both in the field equations for the tensor field and in those for the scalar field; in a weak-field limit these field equations reduce to the equivalent of linear wave equations, but with different values for the rest masses of 'gravitons' and of 'scalarons'. Whereas the former depends algebraically on the value of f_4 in the weak-field limit, the latter also involves the derivative of f_4 with respect to its argument φ. In most discussions the possibility of a cosmological term has been disregarded, on the grounds that there is neither a strong theoretical nor an observational motivation for suspecting that either gravitational or scalar waves propagate with a speed different from the speed of electromagnetic waves.

For a static gravitational field the existence of a cosmological term would lead to a Yukawa-like modification of the inverse-square law; planetary orbits would be the more sensitive to the existence of such an exponential cut-off the larger their dimensions, whereas the well-known relativistic effects are most pronounced for the inner planetary orbits. The absence of any observed deviations from Kepler's Third Law imposes a stringent upper bound on the magnitude of any cosmological term in the equations for the tensor field. There is no comparable accurate information available concerning the properties of the scalar field, if any; hence a cosmological term in the Lagrangian (1.1) cannot be ruled out on observational grounds.

If one were to adopt a Lagrangian of the form

$$L = \sqrt{(-g)}\,[g^{\mu\nu}(R_{\mu\nu} - \varphi_{,\mu}\varphi_{,\nu}) + \tfrac{1}{2}A\varphi^{\mu\nu}\varphi_{\mu\nu} + B] \tag{1.3}$$

with both A and B as yet undetermined functions of φ, the field equations would take the form

$$G_{\mu\nu} - \varphi_{,\mu}\varphi_{,\nu} + \tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma}\varphi_{,\rho}\varphi_{,\sigma} + A(\varphi_{\mu}{}^{\rho}\varphi_{\nu\rho} - \tfrac{1}{4}g_{\mu\nu}\varphi^{\rho\sigma}\varphi_{\rho\sigma}) - \tfrac{1}{2}Bg_{\mu\nu} = 0 \tag{1.4}$$

$$2(g^{\rho\sigma}\varphi_{,\rho})_{;\sigma} + \tfrac{1}{2}\varphi^{\rho\sigma}\varphi_{\rho\sigma}A' + B' = 0, \qquad A' \equiv \frac{dA}{d\varphi}, \qquad B' \equiv \frac{dB}{d\varphi} \tag{1.5}$$

$$-2(Ag^{\rho\sigma}\varphi_{\rho\mu})_{;\sigma} = 0 \tag{1.6}$$

If $A' \neq 0$, then A plays the role of a dielectric factor which, because it is not constant, cannot be eliminated from the theory by a change in the choice of electromagnetic units. As for the wave equation obeyed by the scalar field, (1.5), the inhomogeneous term stemming from the Maxwell term in the Lagrangian is of the third degree in the derivatives of the field variables; hence it will not contribute to the source of the scalar field in the lowest order of a weak-field approximation. But unless $B' = 0$ for $\varphi = 0$, the last term in (1.5) will act as a mass term in the 'scalaron' equation, which in the weak-field approximation will resemble a Klein-Gordon equation.

Both the electromagnetic and the scalar fields act as sources of the gravitational field, but enter equation (1.3) only with terms that are at least of the second degree in these field variables.

In what follows the function $B(\varphi)$ will be set zero, not because in the author's opinion there is any strong physical argument for omitting all cosmological terms, but because in this preliminary and largely speculative study the inclusion of too many possibilities would dissipate the emphasis, which is to be directed toward the degrees of freedom of the scalar field.

2. *Dicke's Theory*

In constructing his own field theory, Dicke (1964) has generally disregarded electromagnetic terms and has set the cosmological term zero. But he has introduced into the Lagrangian an additional term, which was to incorporate, at a non-quantum level, all matter and all fields other than the tensor and the scalar fields. If this term in the Lagrangian be designated by the symbol L_M, then the (variational) derivative of L_M with respect to the metric tensor is to represent the energy-stress tensor density resulting from all contributions except $g^{\mu\nu}$ and φ.

This representation of matter is ordinarily considered a permissible phenomenological simplification, which involves no particular physical assumption. Dicke has, however, incorporated a non-trivial assumption by specifying that in a calibration of the metric and the scalar field in which the usual gravitational term in the Lagrangian is multiplied by the scalar λ and the scalar field term by a specified numerical factor and by λ^{-1}, the matter term is not to depend on the scalar at all. His Lagrangian has the form:

$$L_D = \sqrt{(-g)}\,(\lambda R - (\omega/\lambda)\,g^{\rho\sigma}\lambda_{,\rho}\lambda_{,\sigma}) + L_M \tag{2.1}$$

with

$$\frac{\partial L_M}{\partial \lambda} = 0 \tag{2.2}$$

This requirement can, of course, be translated into the language employed in equations (1.3) through (1.6) (Dicke, 1962). Employing for Dicke's metric tensor the symbol $\bar{g}$ for a moment, the relationships are:

$$g_{\mu\nu} = \lambda \bar{g}_{\mu\nu} \qquad \lambda = \exp[(1/k)\,\varphi] \qquad k = (\omega + \tfrac{3}{2})^{1/2} \tag{2.3}$$

For Dicke to require that L_M be independent of λ is, in the terminology of this paper, equivalent to requiring that L_M depend only on the combination

$$\bar{g}_{\mu\nu} = \exp[-(\varphi/k)]\, g_{\mu\nu} \tag{2.4}$$

If L_M depends on $\bar{g}$ algebraically, then equation (2.2) turns into

$$\frac{\partial L_M}{\partial \varphi} = \frac{1}{k} g^{\mu\nu} P_{\mu\nu}, \qquad P_{\mu\nu} \equiv \frac{\partial L_M}{\partial g^{\mu\nu}} \tag{2.5}$$

That is to say, in terms of the formalism employed in this paper the matter Lagrangian is to furnish a source for the scalar field, but one strictly tied to its gravitational source density, that is the energy-stress tensor. The source density for the scalar field is to be proportional (with the numerical constant k^{-1}) to the trace of the energy-stress tensor. If this principle be applied to the electromagnetic field, reference to equation (1.5) shows that the function $A(\varphi)$ should have to be chosen a constant and, hence, equal to unity by a trivial re-calibration of the electromagnetic variables.

If in Kaluza's time one could argue that a field theory encompassing gravitation and electrodynamics might be 'complete', today's prevailing opinion is that the strong and the weak nuclear forces, and the symmetries discovered in elementary particle research, require that a comprehensive theory of nature allow for additional fields. True, these fields might not allow of a description in the classical (non-quantum) limit. Thus the phenomenological description of these 'miscellaneous' fields and particles might reveal very little about the underlying fundamental theory. But Dicke's assumption, (2.2) or (2.5), applies to the coupling of *all* non-gravitational fields to the hypothetical scalar field at the phenomenological level.

Dicke has justified his assumption by pointing to the experimental verification of the principle of equivalence, which is indeed excellent, partly because of his own contributions. One might, however, draw very different conclusions from the same admitted set of facts. One possible conclusion is that the validity of the principle of equivalence indicates that a scalar field of the Jordan-Thiry type does in fact not exist in nature; if it did, the scalar force should be observed, and it should lead to deviations from the principle of universal free fall, just as electromagnetic coupling does when there are charges.

Another possible conclusion is that the scalar force is not observed ordinarily because the proper conditions have as yet not been provided in high-accuracy experiments. The scalar field might obey a Klein-Gordon equation with non-vanishing rest mass ($B' \neq 0$), and hence lead to a short-range force. Alternatively, the force might be activated only in the case of 'strange' or otherwise exotic particles, whose free fall is difficult to observe. In either case, the scalar field and the resulting forces would probably not have the properties desired by Jordan and by Dicke.

Suppose we adopt the hypothesis of a scalar field but hold Dicke's restrictive assumption in abeyance. According to equation (1.5) the scalar field would propagate as a wave without polarization. Its field equation permits a spherical wave, resembling a spherically symmetric acoustic wave propagating in a fluid. Dipole and other multipole waves are, of course, not excluded. Even if Dicke's assumption were adopted, a spherically symmetric wave can be constructed through the superposition of incoming and outgoing radiation. Without this assumption a purely outgoing wave merely requires a suitable source at the center.

That neither the electromagnetic nor the gravitational fields possess monopole waves is a result of their respective gauge groups, which in turn are required if these fields are to be irreducible. No such symmetry or irreducibility argument applies to scalar fields. Dicke's assumption annihilates an otherwise unexceptionable degree of freedom of the scalar field; neither of the two well-known classical long-range fields is restricted by an analogous condition.

Instead of considering spherical waves, one may also treat slowly variable fields associated with sources. The superselection rules resulting from the respective gauge groups lead to the conservation of electric charge, and to the conservation of mass and linear momentum. Again, no analogous conservation law applies to the source of the scalar field, unless one adopts Dicke's assumption. The source strength of a slowly variable scalar field can be an arbitrary function of the time; in Section 3 it will be shown that this source strength does contribute to the ponderomotive laws. Thus, a lump of matter possessing a variable scalar source strength ('charge') would experience a variable acceleration in the lowest non-trivial approximation of a slow-motion EIH treatment.

I do not consider the present information about the scalar field sufficient to reach firm conclusions either about its existence or about its special properties, particularly its coupling to other fields. The purpose of my discussion is to bring out what I consider open questions and, potentially, grave difficulties for a theory involving a scalar field. In the next section I shall proceed from the assumed field equations of Section 1, with $B = 0$, $B' = 0$, and construct the rigorous ponderomotive theory without Dicke's special assumption. The 'test particle' will have a gravitational mass and an electric charge subject to the (unavoidable) conservation laws, and a scalar field source strength that is an arbitrary function of the time.

3. *Behavior of a Singularity*

Because of the general covariance of the scalar-tensor theory, and because of its gauge invariance, it is possible to obtain statements about the time-dependence of two-dimensional integrals defined on three-dimensional surfaces that fully surround singular world curves (or tubes), in such a manner that on the three-surface itself the vacuum field equations are satisfied. To be non-trivial, such integral

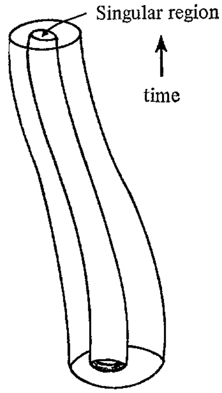

Figure 1.—Shape of integral surface.

relations must be more than identities: Their validity must depend on the field equations being indeed obeyed.

The point of departure for the derivation of such integral relations is the construction of the generators (in the sense of Hamiltonian theory) of infinitesimal coordinate and gauge transformations. These generators satisfy the relationship (Bergmann & Schiller, 1953; Goldberg, 1962):

$$\frac{\delta L}{\delta y^A} \delta' y^A \equiv C^{\rho}{}_{,\rho} \tag{3.1}$$

The expressions $(\delta L/\delta y^A)$ represent the variational derivatives of the Lagrangian with respect to all the field variables, which in the present case are the components of the metric tensor, the electromagnetic potentials, and the scalar potential. The symbols $\delta' y^A$ stand for the infinitesimal transformation laws of the respective field variables under any of the transformations belonging to the invariance group of the theory; they are the appropriate Lie derivatives. The four expressions C^ρ on the right are the components of the generating density. Any integral of the form $\int C^\rho d\sum_\rho$ taken over a space-like three-dimensional hypersurface is, with respect to that surface as an initial hypersurface, the (canonical) generator of the infinitesimal transformation of field variables under the invariance transformation that is being considered. If the invariance transformations of the theory involve arbitrary functions of time, as ours do, then the requirement that the divergence of C^ρ vanish if the field equations

$$\frac{\delta L}{\delta y^A} = 0 \tag{3.2}$$

are satisfied implies that the coefficients of these arbitrary functions in C^ρ can be reduced to zero, though strictly speaking they need only equal the curl of a (four-dimensional) curl. There is no implication that the C^ρ are the components of a vector density, or even that they form a geometric object.

With the Lagrangian (1.4) and the field equations (2.1) through (2.3) the left-hand side of equation (3.1) takes this form:

$$\begin{aligned}\frac{\delta L}{\delta y^A}\delta' y^A &\equiv 2\mathscr{L}^\rho_\mu \xi^\mu{}_{;\rho} + J^\mu[(\xi^5 - \varphi_\rho \xi^\rho)_{;\mu} + \varphi_{\rho\mu}\xi^\rho] - \Phi\varphi_{;\rho}\xi^\rho \\ &\equiv [2\mathscr{L}^\rho_\mu \xi^\mu + (\xi^5 - \varphi_\mu \xi^\mu) J^\rho]_{,\rho} + (\varphi_\mu \xi^\mu - \xi^0) J^\rho{}_{;\rho} \\ &\quad + (\varphi_{\rho\mu} J^\rho - 2\mathscr{L}^\rho_{\mu;\rho} - \Phi\varphi_{,\mu})\xi^\mu\end{aligned} \tag{3.3}$$

From this expression, along with the identity (3.1), one can immediately read off the differential identities obeyed by the field equations. In view of the fact that the right-hand side of (3.3) is, identically, a divergence and that, accordingly, a four-dimensional volume integral, by Gauss's theorem, depends on the (completely arbitrary) values of ξ^ρ, ξ^5 on the boundary of the domain of integration only, the coefficients of the undifferentiated ξ, that is to say the expressions in the last terms, must vanish identically:

$$2\mathscr{L}^\rho_{\mu;\rho} + \varphi_{,\mu}\Phi + \varphi_{\mu\rho}J^\rho \equiv 0, \qquad J^\rho{}_{;\rho} \equiv J^\rho{}_{,\rho} \equiv 0 \tag{3.4}$$

The first set of identities are the analog of the contracted Bianchi identities in the conventional pure gravitational theory, and, of course, they can be verified by direct substitution from equations (2.1) through (2.3). The last identity represents the conservation of electric charge; it forms the basis for a physically reasonable definition of charge-current density, and hence of electric charge, Q, in terms of an integral over a space-like three-dimensional domain or an integral over the closed two-dimensional boundary:

$$Q = -\tfrac{1}{2}\int J^{\rho}\, d\textstyle\sum_{\rho} = -\tfrac{1}{4}\oint |g|^{1/2} A\varphi^{\rho\sigma}\, d\textstyle\sum_{\rho\sigma} \tag{3.5}$$

As for the remainder, the generating density of an infinitesimal invariance transformation turns out to be:

$$C^{\rho} \equiv 2\xi^{\mu}\mathscr{L}^{\rho}_{\mu} + (\xi^{5} - \varphi_{\mu}\xi^{\mu})J^{\rho}, \qquad C^{\rho}{}_{,\rho} = 0, \qquad C^{\rho} = 0 \tag{3.6}$$

These expressions contain derivatives of the field variables up through the second order, which may be eliminated when it is realized that generating densities are defined by the relationship (3.1) only up to the addition of a curl. By making the substitution (Bergmann, 1958; Goldberg, 1958)

$$2|g|^{1/2} G^{\rho}_{\mu} \equiv \mathscr{A}^{\rho}_{\mu} - U^{\rho\sigma}{}_{\mu,\sigma} \tag{3.7}$$

where $\mathscr{A}^{\rho}_{\mu}$ are Einstein's canonical expressions for the energy-stress complex, and $U^{\rho\sigma}{}_{\mu}$ are Freud's expressions (von Freud, 1939), which are free of second derivatives, and linear in the first derivatives of the metric tensor, one can obtain the following equation of continuity, instead of the second equation (3.6):

$$\begin{aligned} \bar{C}^{\mu} = {} & \mathscr{A}^{\mu}_{\rho}\xi^{\rho} + U^{\mu\sigma}{}_{\rho}\xi^{\rho}{}_{,\sigma} + |g|^{1/2}\varphi_{,\rho}(\xi^{\mu}\varphi_{,}{}^{\rho} - 2\xi^{\rho}\varphi_{,}{}^{\mu}) \\ & + 2|g|^{1/2} A[\varphi_{\rho\sigma}\varphi^{\mu\sigma}\xi^{\rho} - \tfrac{1}{4}\varphi^{\rho\sigma}\varphi_{\rho\sigma}\xi^{\mu} - 2\varphi_{\rho,\sigma}\varphi^{\mu\sigma}] \\ & - 2|g|^{1/2} A\varphi_{\rho}\varphi^{\mu\sigma}\xi^{\rho}{}_{,\sigma}, \end{aligned} \tag{3.8}$$

$$\bar{C}^{\mu}{}_{,\mu} = 0$$

The $\bar{C}^{\mu}$ are entirely free of second-order derivatives. They do not vanish, but their divergence does if or where the field equations are satisfied.

The $\bar{C}^{\mu}$ may be cast into the form of a four-dimensional curl,

$$\bar{C}^{\mu} = [\xi^{\rho}(U^{\mu\sigma}{}_{\rho} - 2|g|^{1/2}\varphi_{\rho} A\varphi^{\mu\sigma})] \tag{3.9}$$

Now Stokes's theorem becomes applicable. Given a three-dimensional 'sleeve' surrounding a singular world tube, as sketched in Fig. 1, we have

$$\int \bar{C}^{\mu}\, d^{3}\textstyle\sum_{\mu} = \oint \xi^{\rho}(\tfrac{1}{2}U^{\mu\sigma}{}_{\rho} - |g|^{1/2} A\varphi_{\rho}\varphi^{\mu\sigma})\, d^{2}\textstyle\sum_{\mu\sigma} \tag{3.10}$$

If the 'sleeve' is permitted to contract lengthwise into an infinitesimal band whose (coordinate) width equals dt, then equation (3.10) turns into the condition of time dependence:

$$\oint \bar{C}^{k} d^{2}S_{k} = \frac{d}{dt}\oint \xi^{\rho}(U^{k0}{}_{\rho} - 2|g|^{1/2}\varphi_{\rho}\varphi^{k0})\, d^{2}S_{k} \tag{3.11}$$

Equations (3.10), (3.11) are associated with the coordinate invariance of the action principle. As for the gauge invariance, the expressions are very simple. The generating density that is free of second-order derivatives is:

$$\bar{C}^{\rho} \equiv 2|g|^{1/2} A\xi^{5}{}_{,\tau}\varphi^{\rho\tau} = 2(|g|^{1/2} A\varphi^{\rho\tau}\xi^{5})_{,\tau} \tag{3.12}$$

It follows that

$$\int \bar{C}^{\rho} d^3 \textstyle\sum_{\rho} = \oint |g|^{1/2} A\xi^{5}\varphi^{\rho\tau} d^2 \textstyle\sum_{\rho\tau} \tag{3.13}$$

and

$$\oint |g|^{1/2} A\xi^{5}{}_{,\tau}\varphi^{k\tau} d^2 S_k = \frac{d}{dt}\oint |g|^{1/2} A\xi^{5}\varphi^{k0} d^2 S_k \tag{3.14}$$

This relationship expresses the law of conservation of electric charge in integral form. As the function ξ^5, which appears both on the left and on the right, is arbitrary, a relationship such as (3.14) may be decomposed into an infinite but discrete set of relationships, each of which represents a particular choice for that function. For instance, one might restrict one's choice to such functions whose time derivative vanishes, so that the left-hand side involves only magnetic but not electric components of the field, but choose as regards space-dependence spherical harmonics, performing the integrals on the unit sphere. One will then find that the time derivatives of all the surface integrals over the electric displacement, weighted with the various spherical harmonics, are given by surface integrals involving the cross product of the magnetic induction by the gradient of the same spherical harmonic. Choosing for ξ^5 a constant one obtains on the right the time derivative of the charge, and on the left zero.

Similarly, the relationship (3.11) may be decomposed into an infinity of separate integral conditions, depending on the choices of the arbitrary functions ξ^{ρ}. Again, the time dependence yields only identical terms on the left and on the right, and one exhausts the variety of non-trivial relations by restricting oneself to functions that are constant in time. If in a weak-field approximation the weighting functions are set constant, one obtains on the right expressions that might be interpreted as energy and linear momentum, corrected for the interaction of the electric charge-current with the external potential. The time derivatives of these integrals are given by the integral over Poynting's vector and by the force, respectively, the latter also represented as a surface integral.

It is significant that the scalar field enters on the left of (3.11) in the form of terms that are quadratic in its gradient, but that it enters on the right only algebraically, by way of the function A, which represents the (variable) dielectric 'constant' of the vacuum. Otherwise the (canonical) mass, and the (canonical) linear momentum, of a piece of matter, those quantities that are subject to dynamic laws, depend primarily on metric and electromagnetic variables. They do not involve the gradient of the scalar field. That this gradient field appears on the left proves that it does contribute to the force, and to Poynting's vector, quadratically.

In the weak-field case, in order to determine the motion of a test particle in an external field, one calculates the mass and the linear momentum in linear approximation, obtaining the mass, for instance, as the integral over the gradient of the Newtonian potential. In this approximation the 'mass' depends entirely on metric components, and not at all on either electromagnetic or scalar-field contributions. As the leading terms on the left are quadratic in the various field strengths, the important terms to be taken into consideration in the lowest-order approximation are bilinear in contributions by the self-field of the particle and by the external field. Thus one obtains, in a very intuitive fashion, such products as mass times gravitational field strength, electric charge times electric field strength, and finally scalar source strength times scalar field strength.

As mentioned earlier, the scalar source strength differs from mass and electric charge in that the latter two properties of a test particle are in lowest order constants of the motion, whereas the scalar source strength is an arbitrary function of time. Hence, even if the external field is given by the distribution of the large astronomical objects, the force acting on a test particle contains one term which even in the lowest-order approximation involves an arbitrary function of time. Hence the motion of a test particle in an external field is undetermined in the scalar-tensor theory, in gross contradiction to experimental evidence.

4. *Concluding Remarks*

The principal point of this paper is a discussion of Dicke's assumption, which restricts the role of non-gravitational fields as sources of the scalar field. Whereas in other respects the scalar field enters his theory as one of the three classical long-range fields, he subjects its lowest-order spherical normal mode to conditions that have no analogs in contemporary theory.

Without wishing to imply that this facet of his theory should be discarded out of hand, I have constructed the rigorous ponderomotive laws of a scalar-tensor theory in which 'matter' is represented by separable regions in space in which the vacuum field equations do not hold, without any further conditions. The integral laws (3.11) and (3.14) hold irrespective of any weak-field or slow-motion approximations. Their explicit forms show that the mass and the linear momentum attributable to a separable lump of matter are defined without reference to the scalar-field gradient, and that the rates of change of these quantities depend, *inter alia*, on that gradient. Hence any time dependence of the scalar-field source strength associated with a lump of matter will affect its ponderomotive behavior.

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